half life style questions

1. Medical researchers studying the growth of a strain of bacteria observe that the number of bacteria, present after t hours, is given by the formula $N(t) = 40e^{1.5t}$.

(a) State the number of bacteria present at the start of the experiment.   1

(b) How many minutes will the bacteria take to double in number?   4

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	C	CR	A6		1989 P1 Q20
(b)	1	C	CR	A30		
(b)	3	A/B	CR	A30		

- $\bullet^1$ 40
- $\bullet^2$ $40e^{1.5t} = 80$
- $\bullet^3$ $1.5t = \ln 2$
- $\bullet^4$ $t = 0.46$
- $\bullet^5$ 28 minutes

2. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k.   3

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CR	A30	$k = 0.46$	2001 P2 Q9

- $\bullet^1$ ic: form exponential equation
- $\bullet^2$ ss: express exp.   equ.   as log equation
- $\bullet^3$ pd: solve log equation

- $\bullet^1$ $2A_0 = A_0 e^{k \times 1.5}$
- $\bullet^2$ e.g. $1.5k = \ln 2$
- $\bullet^3$ $k = 0.46$

3. The amount A grams of a radioactive substance at time t minutes is given by $A = A_0 e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.

In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(a) Find the value of k.   3

The half-life of a substance is the length of time in which half the substance decays.

(b) Find the half-life of Bismuth.   2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	C	CR	A30		1995 P1 Q18
(a)	2	A/B	CR	A30, A34		
(b)	2	A/B	CR	A30		

- $\bullet^1$ $9 = 10e^{-3k}$
- $\bullet^2$ $-3k = \log_e 0 \cdot 9$
- $\bullet^3$ $0 \cdot 04$
- $\bullet^4$ $e^{-kt} = 0 \cdot 5$
- $\bullet^5$ a correct value for t

4. A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_c is the temperature after t minutes. All temperatures are in °C.

(a) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k.   3

(b) By how many degrees will the temperature of this tea fall in the next quarter of an hour?   2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	C	CR	A30		1996 P1 Q19
	4	A/B	CR	A30, A34		

- $\bullet^1$ $75 = 100e^{-k \times 15}$
- $\bullet^2$ $\ln 0 \cdot 75 = -15k$
- $\bullet^3$ $k = 0.0192$
- $\bullet^4$ $T_{15} = 75e^{-0.0192 \times 15}$ or $T_{30} = 100e^{-0.0192 \times 30}$
- $\bullet^5$ fall = 18.75

5. (a) A tractor tyre is inflated to a pressure of 50 units.
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places. **(5)**

(b) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre? **(4)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	C	CR	A30, A34		1991 P2 Q7
(a)	3	A/B	CR	A30, A34		
(b)	1	C	CR	A30		
(b)	3	A/B	CR	A34		

(a)
- $\bullet^1$ $10 = 50e^{-24k}$
- $\bullet^2$ $0.2 = e^{-24k}$
- $\bullet^3$ $-24k = \ln 0.2$
- $\bullet^4$ $-24k = -1.609$
- $\bullet^5$ $k = 0.067$

(b)
- $\bullet^6$ knowing to find P_4
- $\bullet^7$ $P_4 = 50e^{-0.067 \times 4}$
- $\bullet^8$ 38
- $\bullet^9$ $38 > 30$ so can be driven further

6. (a) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years. **(2)**

(b) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance. **(3)**

(c) Illustrate **ALL** of the above information on a graph. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	C	CR	A30, A34		1992 P2 Q4
(b)	1	C	CR	A30		
(b)	2	A/B	CR	A30, A34		
(c)	1	C	CR	A2		
(c)	2	A/B	CR	A2		

(a) •^{1} $m = 500e^{-0.02 \times 10}$
     •^{2} 409.37 grams

(b) •^{3} $250 = 500e^{-0.02t}$
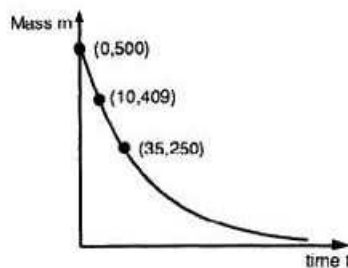   •^{4} $\ln 250 = \ln 500 - 0.02t \times 1$ or equiv.
     •^{5} 34.7 years

(c) •^{6} any **two** of the 3 points
     •^{7} the remaining point
     •^{8} a decreasing curve

Mass m
(0,500)
(10,409)
(35,250)
time t

7. The size of the human population, N, can be modelled using the equation $N = N_0 e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

    (*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020?   **2**

    (*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

    Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size?   **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	B	CR	A30, A34	76 million	2009 P2 Q6
(*b*)	3	A	CR	A30, A34	$t = 161 \cdot 2$ years	

$\bullet^1$ ic: substitute into equation	$\bullet^1$ $61e^{0 \cdot 016 \times 14}$
$\bullet^2$ pd: evaluate exponential expression	$\bullet^2$ 76 million
$\bullet^3$ ic: interpret info and substitute	$\bullet^3$ $10 \cdot 2 = 5 \cdot 1 e^{0 \cdot 0043 t}$
$\bullet^4$ ss: convert expo. equ. to log. equ.	$\bullet^4$ $0 \cdot 0043 t = \ln 2$
$\bullet^5$ pd: process	$\bullet^5$ $t = 161 \cdot 2$ years

8. The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(a) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(b) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	C	CR	A30, A34		1997 P2 Q8
(a)	2	A/B	CR	A30, A34		
(b)	3	A/B	CR	A30		

(a) • ^{1} $\frac{1}{2} y_0 = y_0 e^{5700k}$

• ^{2} $\ln \frac{1}{2} = 5700k$

• ^{3} $k = -0.000122$

(b) • ^{4} $y = y_0 e^{-0.000122 \times 1000}$

• ^{5} $\frac{y}{y_0} = \ldots$

• ^{6} 88.5%

9. The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(a) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k.

4

(b) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity.

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	C	CR	A30, A34		1999 P2 Q7
(a)	2	A/B	CR	A30, A34		
(b)	1	C	CR	A30		
(b)	2	A/B	CR	A30		

(a) $\bullet^1$ $90 = 120e^{-4k}$

$\bullet^2$ $e^{-4k} = 0.75$ **or** $\ln 90 = \ln 120 + \ln e^{-4k}$

$\bullet^3$ $\ln 0.75 = -4k$

$\bullet^4$ $k = 0.0719$

(b) $\bullet^5$ $I_{10} = I_0 e^{-10 \times 0.0719}$ stated or implied by $\bullet^6$

$\bullet^6$ $\frac{I_{10}}{I_0} = 0.487$

$\bullet^7$ 51.3% reduction

[END OF QUESTIONS]